TABLE I

F_0(GHz)	L_0(dB)	L_{min}(dB)	L_{max}(dB)	$L_{0'} = \frac{L_{max} + L_{min}}{4}$	$L_F = \frac{L_{max} - L_{min}}{2}$	$L_C = L_{0'} - L_F$	Ferrite Material (Trans-Tech)
4.4	0.16	0.24	0.37	0.15	0.065	0.085	G1004
4.6	0.14	0.25	0.36	0.152	0.055	0.097	G1004
4.8	0.15	0.26	0.34	0.147	0.040	0.107	G1004
9.0	0.18	0.32	0.44	0.19	0.06	0.13	TT1-390
10.5	0.186	0.29	0.47	0.192	0.09	0.101	TT1-390
9.0	0.17	0.31	0.37	0.172	0.03	0.14	G1001

eigennetwork, and furthermore assume that the dissipation of the two counterrotating eigennetworks is equal.

Substituting the preceding relations into (1) gives

$$L = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] - L_0\left(1 - \frac{L_0}{2}\right)\cos\left(\Psi_1 + \Psi_2\right) \tag{7}$$

where we have put

$$X = L_0.$$

The theoretical minimum and maximum values of the loss are therefore

$$L_{min} = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] - L_0\left(1 - \frac{L_0}{2}\right) \tag{8}$$

$$L_{max} = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] + L_0\left(1 - \frac{L_0}{2}\right). \tag{9}$$

For instance, when $L_0 = 0.50$ dB the result is

$$L_{min} = 0.475 \text{ dB}$$

$$L_{max} = 1.445 \text{ dB}.$$

Equations (8) and (9) suggest that for L_0 small L varies between L_0 and $3L_0$.

The following results have been obtained for a waveguide circulator consisting of a simple lossy ferrite post at the junction of three rectangular waveguides:

$$L_0 = 0.36 \text{ dB}$$

$$L_{min} = 0.38 \text{ dB}$$

and

$$L_{max} = 1.14 \text{ dB}.$$

In the presence of circuit losses the following empirical relations apply:

$$L_{min} \approx 2(L_C + L_F) - L_F \tag{10}$$

$$L_{max} \approx 2(L_C + L_F) + L_F \tag{11}$$

where L_F is the single path ferrite loss and L_C is the single path circuit loss. The first term in the preceding two equations represents twice the single path loss which is consistent with the observation in [5].

This experiment may therefore be employed to separate ferrite and circuit losses in 3-port junction circulators—some experimental results which apply to quarter-wave coupled waveguide circulators at C and X bands are given in Table I. The circuit losses obtained here are consistent with those given in [4].

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Application of a Property of the Airy Function to Fiber Optics

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***Abstract*—The integral of the square of the Airy function from one of its zeros to infinity is equal to the square of the first derivative of the Airy function at the zero considered. Two important applications of this result to fiber optics are discussed.**

The Airy function is involved in many problems of fiber optics. For example, waves guided along the curved boundary of a homogeneous dielectric [1] (whispering gallery modes [2]) or along the straight boundary of a medium with constant transverse gradient of refractive index [3], are described by Airy functions. We shall show that the normalized field at the dielectric boundary is given by a very simple expression because of a property of the Airy function that does not seem to be known. Knowledge of the normalized field is essential to evaluate the coupling strength and the bending loss of a mode.

The Airy function Ai(x) is a solution of the differential equation [4]

$$d^2\,\text{Ai}\,(x)/dx^2 = x\,\text{Ai}\,(x). \tag{1}$$

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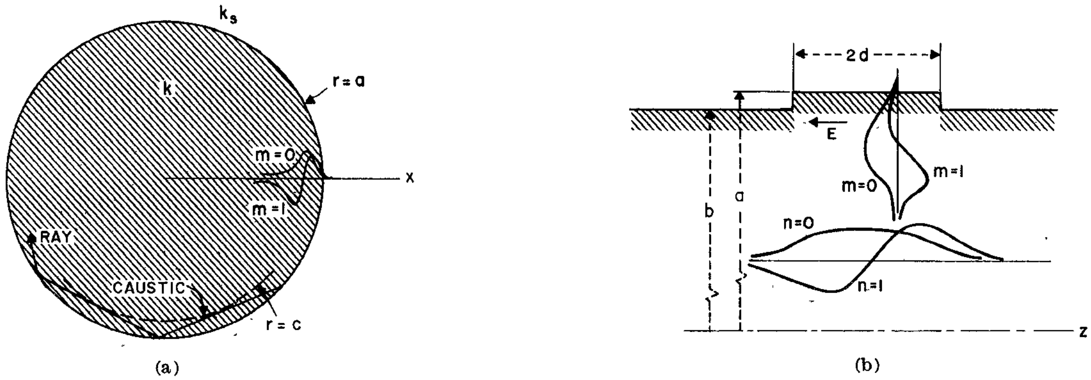

Fig. 1. (a) Represents a cross section of the dielectric rod. The mode field has an oscillatory behavior between the caustic with radius c, and the rod boundary with radius a. The field decays exponentially in the surrounding medium. (b) The wave can be kept confined in the axial (z) direction by a slight reduction of the rod radius, from $r = a$ to $r = b$.

Let us evaluate the integral

$$I = \int_x^\infty \mathrm{Ai}^2(x)\, dx. \tag{2}$$

Integrating by parts, we have

$$I = x\,\mathrm{Ai}^2(x) \Big|_x^\infty - 2\int_x^\infty x\,\mathrm{Ai}(x)[d\,\mathrm{Ai}(x)/dx]\,dx. \tag{3}$$

If we use the differential equation (1), (3) can be written

$$\begin{aligned} I &= x\,\mathrm{Ai}^2(x) \Big|_x^\infty - 2\int_x^\infty [d\,\mathrm{Ai}(x)/dx][d^2\mathrm{Ai}(x)/dx^2]\,dx \\ &= x\,\mathrm{Ai}^2(x) \Big|_x^\infty - [d\,\mathrm{Ai}(x)/dx]^2 \Big|_x^\infty \\ &= -x\,\mathrm{Ai}^2(x) + [\mathrm{Ai}'(x)]^2 \end{aligned} \tag{4}$$

where

$$\mathrm{Ai}'(x) \equiv d\,\mathrm{Ai}(x)/dx$$

because lim $(x \to \infty)$ of $x\,\mathrm{Ai}^2(x) = 0$, and lim $(x \to \infty)$ of $\mathrm{Ai}'(x) = 0$. If $x = x_\alpha$ is a zero of the Airy function, the simple result

$$\int_{x_\alpha}^\infty \mathrm{Ai}^2(x)\, dx = [\mathrm{Ai}'(x_\alpha)]^2 \tag{5}$$

is obtained.

As a first example of application of (5), let us consider whispering gallery modes guided along the circular boundary of a dielectric rod, with radius a. The number of plane waves in the dielectric material is denoted k. The wavenumber of plane waves in the surrounding medium (or cladding) is denoted k_s. We assume that the ratio k/k_s is not very different from unity and make the scalar approximation.

The field of whispering gallery modes has the form

$$\psi(x) = \psi_0\,\mathrm{Ai}[\kappa(-x - a + c)], \qquad x < 0 \tag{6}$$

where Ψ_0 is a constant and

$$\kappa = 2^{1/3}k^{2/3}c^{-1/3} \tag{7}$$

as we can see by taking the asymptotic form of Bessel's functions. In (6) and (7), c denotes the caustic radius, a quantity that we shall define later, and $x \equiv r - a$ [see Fig. 1(a)]. The azimuthal wavenumber is equal to k at the caustic radius c, and therefore, it is equal to $(c/a)k$ at the rod radius a. The field outside the rod is approximately given by an exponential

$$\begin{aligned} \Psi(x) &\approx \exp(-sx), \qquad x > 0 \\ s &= (k^2c^2/a^2 - k_s^2)^{1/2}. \end{aligned} \tag{8}$$

Continuity of the field and of its first derivative at the rod boundary $r = a$ (or $x = 0$) requires that from (6) and (8)

$$\begin{aligned} \psi_0\,\mathrm{Ai}[\kappa(-a + c)] &= 1 \\ \psi_0\,\kappa\,\mathrm{Ai}'[\kappa(-a + c)] &= s. \end{aligned} \tag{9}$$

The caustic radius c is now defined by (9).

The power of the mode is proportional to the integral of $k\psi^2(x)$ from $x = -\infty$ to the rod boundary, p is the integral of $k_s\psi^2(x)$ from the rod boundary to $x = +\infty$. Using (6), (8), (9), and the result in (4) we obtain

$$\begin{aligned} P &= k\int_{-\infty}^0 \psi_0^2\,\mathrm{Ai}^2[\kappa(-x - a + c)]\,dx + k_s\int_0^\infty \exp(-2sx)\,dx \\ &= ks^2/\kappa^3 + k(a - c) + k_s/2s. \end{aligned} \tag{10}$$

For large values of the rod normalized frequency $F \equiv (k^2 - k_s^2)^{1/2}a$ (sometimes denoted V), and low-order modes, the wave clings tightly to the boundary ($c - a \ll a$) and the field at the boundary is very small compared with the field inside the rod in the annular region $c < r < a$. Thus, in the limit $F \to \infty$, the square of the normalized field $\hat{\psi}^2 = \psi^2/P$ at the rod boundary is obtained by neglecting the last two terms in (10), using (7) and the approximation $s \approx (k^2 - k_s^2)^{1/2}$. The result is

$$\hat{\psi}^2 \equiv \psi^2/P \approx \kappa^3/ks^2 = 2k/(k^2 - k_s^2)a. \tag{11}$$

It is remarkable that this simple result does not involve the Airy function or its zeros. The radiation leak of whispering gallery modes is easily obtained from this expression in (11) and the general formulas in [5]. (For a comparison see [6]).

Whispering gallery modes can be kept confined in the axial (z) direction if the rod radius is reduced to a slightly smaller radius b on both sides of the central region as shown in Fig. 1(b). The azimuthal wavenumbers of the trapped modes (with mode indices m, n in the radial and axial directions, respectively) are calculated in [7] by two different methods. First, by matching the azimuthal wavenumbers at the junction of the central region, of width $2d$ and radius a, and at the outer regions of radii b, and secondly, by a perturbation method. The results obtained from these two methods were thought to agree closely but not exactly. The result in (5) of the present short paper shows that the agreement between the two methods is, in fact, exact. The ratio of the right-hand side to the left-hand side of (5) was inaccurately given in [7] as 0.981 for the first zero (fundamental Airy mode) and 0.955 for the second zero. We now recognize that this ratio is unity for all the zeros.

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The Relation of Teratogenesis in *Tenebrio molitor* to the Incidence of Low-Level Microwaves

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***Abstract*—The teratogenic effects of irradiation by low-level microwaves have been studied using the pupae of the darkling beetle *Tenebrio molitor*. For exposures of 2-h duration, statistically significant increases in teratogenesis were observed at waveguide power levels down to 200 μW; the pupation time increased monotonically with the power. Exposures of various durations and powers at a constant dosage of 4 mW/h strongly suggested that it is the total dosage which determines the level of teratological damage.**

I. INTRODUCTION

Lindauer *et al.* [1] have reported: 1) that statistically significant teratological damage can be inflicted upon the pupae of the darkling beetle *Tenebrio molitor* by microwave irradiation at 9 GHz and power level of as little as 8.6-mW/cm² CW (10-mW level in WR-90 waveguide) for 2-h exposures; and 2) that there is no significant difference between exposure at 20 mW for 2 h and exposure at 10 mW for 4 h. These observations raised two questions. First, what is the minimum power level which will, with 2-h exposures, have a statistically significant teratogenic effect? Second, since the response of a biological system to a stimulus is often a function of the product of the stimulus intensity and the exposure time, do the results of Lindauer *et al.* [1] indicate the existence of such a reciprocity relation? The experiments described in the following were carried out to answer these questions.

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II. MATERIAL AND METHODS

One- to two-day-old pupae (nominally, 15-mm length and 5-mm diameter and cultured as reported previously [1]) were mounted for irradiation in Styrofoam blocks and then inserted along the center line of an X band waveguide with their anteriors toward the power source. The experimental arrangement and the microwave circuit for irradiating pupae were the same as those described by Lindauer *et al.* [1], to which the reader is referred for full details and a schematic of the apparatus. The operating frequency was again 9 GHz. The pupae for the control group were mounted in waveguides as if for irradiation but no microwave power was applied. The irradiated pupae and control pupae were placed in individual numbered vials in a darkened environmental chamber at 21°C for the duration of pupation. Using "single blind" techniques, the emergent adults were categorized for gross morphological defects according to the scheme of Carpenter and Livstone [2] where

- D insect died during pupation;
- $G1$ insect developed head and thorax of an adult, but retained the abdomen of a pupa, sometimes with pupal case attached;
- $G2$ adult insect had rumpled and grossly distorted elytra and/or shredded wings;
- $G3$ adult insect was normal except for small discrete holes in elytra;
- N adult insect was apparently normal.

To determine the effects of power level at constant duration of irradiation, pupae were irradiated for 2 h at different levels of incident power, the level being reduced until statistically significant damage was no longer observable with the number of the pupae employed. Pupae absorb roughly $\frac{1}{3}$ of the incident power [1]. The total incident power (in milliwatts) can be converted to power per unit area (in milliwatts per square centimeter) at the center of the WR-90 waveguide employed by multiplying by 0.85; thus, at a level of 20 mW a pupa is exposed to a power density of 17 mW/cm² and is absorbing energy at a rate of roughly 7×10^{-3} J/s.

To determine the effects of power level at constant dosage, pupae were given exposures of 4 mW/h (*e.g.*, 2 mW of CW waveguide power for an uninterrupted 2-h exposure) at power levels of 2^{n-1} mW ($0 \leq n \leq 5$) and corresponding exposure times of 2^{3-n} h.

III. RESULTS AND STATISTICAL ANALYSIS

The results of irradiation at various power levels are shown in Table I. These data were analyzed by the classic chi-square test [3] with categories $G1$, $G2$, and $G3$ combined to avoid undue emphasis

TABLE I
INCIDENCE OF TERATOGENIC DAMAGE FOR 2 H OF EXPOSURE AT VARIOUS POWER LEVELS

Group	D	G1	G2	G3	N	Total
20 mW	20 (26.7%)	11 (14.6%)	23 (30.7%)	7 (9.3%)	14 (18.7%)	75
10 mW	21 (26.7%)	10 (12.5%)	20 (25.0%)	5 (6.3%)	24 (30.0%)	80
2 mW	16 (23.9%)	6 (9.0%)	14 (20.9%)	2 (3.0%)	29 (43.3%)	67
1 mW	17 (23.3%)	6 (8.2%)	15 (20.5%)	2 (2.7%)	33 (45.2%)	73
0.2 mW	14 (20.9%)	5 (7.5%)	8 (11.9%)	1 (1.5%)	39 (58.2%)	67
0.1 mW	21 (20.0%)	7 (6.7%)	11 (10.5%)	1 (1.0%)	65 (61.9%)	105
0.05 mW	11 (16.2%)	4 (5.9%)	6 (8.8%)	0 (0.0%)	47 (69.1%)	68
Control	54 (18.1%)	13 (4.4%)	22 (7.4%)	1 (0.3%)	208 (69.8%)	298